Guidelines to Developing Software for Thermoeconomic Analysis of Energy Systems.

Part I: The Thermoeconomic Model

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ABSTRACT: Thermoeconomics, an energy analysis methodology combining thermodynamics and economics, has been used for more than twenty years to support the design, synthesis and operation of energy systems with higher efficiency and lower unit production costs. There is a wide range of computer applications for modelling, process integration, simulation, optimisation and control of energy systems, but in general, they don't include thermoeconomic analysis. The paper, divided in two parts, should be understood as a review of the fundamental concepts of thermoeconomics, which serves as a functional description of requirements to develop software for thermoeconomic analysis and its integration with existing computer aided applications for energy systems. The first part of the paper focuses on the definition of a thermoeconomic data model and the algorithms that allow one to build an interface to communicate with other energy analysis applications.

Keywords: Thermoeconomics, Computed-aided engineering of energy systems.

NOMENCLATURE

n	Number of components
m	Number of flows
E	Exergy of a flow [kW]
F	Fuel exergy of a component [kW]
P	Product exergy of a component [kW]
E^*	Exergy Cost [kW]
c	Unit exergy cost [kW/kW]
y	Exergy distribution ratios
v	Node of a graph
e	edge of a graph

Matrices and Vectors

$\mathbf{A}$	Incidence matrix (n × m)
$\mathbf{A}_F$	Fuel incidence matrix (n × m)
$\mathbf{A}_P$	Product incidence matrix (n × m)
$\boldsymbol{\alpha}_e$	Resource incidence matrix (ν_e × m)
$\boldsymbol{\alpha}_x$	Bifurcation incidence matrix (ν_x × m)
$\boldsymbol{\omega}_e$	Resource vector (ν_e × 1)
$\mathbf{E}$, $\mathbf{E}^*$	Exergy and Exergy cost vectors (m × 1)
$\mathbf{F}$, $\mathbf{P}$	Fuel and Product vectors (n × 1)
$\langle\mathbf{FP}\rangle$	Distribution ratios matrix (n × n)

Greek letters

ν_e	Number of system inputs
ν_x	Number of bifurcations

Sets

$\mathcal{V}$	Nodes or components of a system
$\mathcal{E}$	Edges of flows of a system
$\mathcal{F}_i$	Fuel streams of a component
$\mathcal{P}_i$	Fuel streams of a component

Subscripts and superscripts

e	related to system inputs
(-1)	Generalized inverse

1. INTRODUCTION

Thermoeconomics, coined by M. Tribus and R. Evans at UCLA in the early 60's, is a combination of the terms thermodynamics and economics. It combines these two disciplines for the design, optimization and operation of energy systems. See references [1]-[5].

In thermodynamic analysis, the study is generally focused on describing the processes and relationships between mass flow streams and energy exchanges. The comparison between alternatives for the design characteristics of the equipment is usually made by thermodynamic measures such as the efficiency or irreversibility rates. On the other hand, economic considerations require a proper balance between thermodynamic efficiency and capital expenditures to achieve a minimum unit production cost.

In thermoeconomics, exergy is often used as a cost carrier because it is a sensitive magnitude to the changes of quality and quantity of the energy processed and as consequence more appropriate than energy to measure the system efficiency.

Some basic and well known concepts should be considered to develop software applications for thermoeconomic analysis. Energy systems represent complex networks of mass and energy flows. Depending on the type of analysis carried out, the system could have a different level of aggregation. The total system is composed of a collection of subsystems. Each subsystem can be a part of a physical device, the device itself or a group of devices.

For the synthesis and design of such systems, it is highly desirable to have a flexible tool for selecting the adequate configuration that requires modular software in the form of building blocks. The philosophy of building blocks and its application to energy systems analysis led to the concept of flow sheeting [6]. In this approach several blocks or processes are connected into an overall plant design. Heat, work and mass flow streams are connected to various blocks in the appropriate way. The blocks must be classified by the type of thermodynamic process performed: heat exchange, compression, expansion, mixing,… ASPEN PLUS [7] is a reference example of the building blocks philosophy.

Due to the complexity of the systems, the sequential-modular approach, which is the way that classical thermodynamics problem have been solved, may not be possible. Therefore, a more general approach is required, which is based on the simultaneous solving of the mass, energy, exergy and cost balances for every system component, according to the block connectivity information. Thermodynamic, thermophysical and chemical properties of the flows are required to determine these equations for a sufficient degree of accuracy over a wide range of operating conditions. It must include the exergy computation of all mass and energy streams. For example, ExerCom [8] is a plug-in integrated with ASPEN which computes the exergy values of all streams of the plant. Unfortunately there are not many building block applications [9] that provide exergy calculations.

To illustrate the proposed methodology, we will use a simple example of a gas turbine cogeneration plant, called TGAS, which is described in reference [10]. Its flow sheet diagram is depicted in Figure 4.

2. THE THERMOECONOMIC MODEL

To provide the thermoeconomic analysis of an energy system, a set of information that constitutes the *thermoeconomic model* of an energy system is required. What follows is a reminder of the *very well known basis* of thermoeconomics, published elsewhere, and selected primarily to look at what is important for defining a functional description of requirements for developing thermoeconomics software.

Physical Structure

A suitable data structure representation of an energy system, for a specific level of aggregation, is made by means of a directed graph, consisting of:

- A set $\mathcal{V} = \{v_0, v_1, \ldots, v_n\}$ of graph nodes that represents the n components of the system. The node v_0 represents the system environment.
- A set $\mathcal{E} = \{e_1, \ldots, e_n\}$ of graph edges, where each element e=(u, v) is an ordered pair of distinct nodes, which represents the mass and energy flows of the systems.

Each component v_i has attributes to describe it, such as a name, type of thermodynamic process and other specific parameters.

Each flow e_i is defined by the pair of components that it links, and other attributes such as the type of flow: mass, heat, work, waste,… could be included.

The *incidence* matrix function defines how the flows and components are connected:

$$\mathbf{A}(i,j)=\begin{cases}+1 & \text{if } e_j=(v_k,\mathbf{v}_i)\\ -1 & \text{if } e_j=(\mathbf{v}_i,v_k)\\ 0 & \text{otherwise}\end{cases}$$

each element $\mathbf{A}(i,j)$ takes the value +1 if the flow e_j enters component v_i, −1 if leaves it, and 0 if there is no relation between them.

The incidence matrix allows us to express the mass, energy, exergy and cost balances. This matrix approach significantly helps the software implementation, and we follow this philosophy in the rest of the paper.

Thermodynamic model

The thermodynamic model (TDM) of the plant is described through a set of equations, (including the mass, energy and entropy balances) that allows every mass flow stream and every heat and work interactions involve in the physical structure of the plant to be determined from a set of input variables, that defines a *thermodynamic state* of the plant.

To carry out the thermoeconomic analysis of a system, it is necessary to provide the exergy E_i of each flow defined in the physical structure. Following an Object Oriented Programming (OOP) approach, the given energy system represents an object and the thermodynamic state is an instance of this object.

Economic model

Moreover, a economic model could be provided that includes:

- The investment and operational cost of the equipment (€/h), defined as a function of a list of parameters, including size, materials, operating range, working hours per year, inflation rates, installation and maintenance cost, as well as other factors.
- The market prices of the fuels (c/kWh), natural gas, coal and other external resources.

Productive structure

Each component or process of the system has a productive purpose. It is established by means of the definition of its efficiency:

Efficiency = Product / Resources

that measures the quality of a process. This is to say, there is an implicit classification of the flows crossing the boundary of the system, the flows that are the production objective, and those that are the resources required to carry out the production.

For each process or component of the system is necessary to identify the flow streams that constitute its *product streams*, and the flow streams used to obtain them, called *fuel streams*. Accordingly, it can be said that the *fuel* is the amount of exergy provided by the fuel streams, and the *product* is the exergy provided by the product streams.

To describe the *productive structure* we must identify, for every component, one or several fuel and product streams as the collection of the flows that constitute them.

We must also provide an incidence matrix for the fuel defined by:

$$\mathbf{A}_{\mathrm{F}}(i,j)=\begin{cases}\mathbf{A}(i,j) & \text{if } e_j \in \mathcal{F}_i\\ 0 & \text{otherwise}\end{cases}$$

where $\mathcal{F}_i$ is the set of fuel streams of the v_i component. And a product incidence matrix:

$$\mathbf{A}_{\mathrm{P}}(i,j)=\begin{cases}-\mathbf{A}(i,j) & \text{if } e_j \in \mathcal{P}_i\\ 0 & \text{otherwise}\end{cases}$$

where $\mathcal{P}_i$ is the set of product streams of the v_i component, which verify: $\mathbf{A}=\mathbf{A}_{\mathrm{F}}-\mathbf{A}_{\mathrm{P}}$.

These incidence matrices allow the computation the exergy of the fuel and product of each component as:

$$\begin{aligned}\mathbf{A}_{\mathrm{F}}\cdot\mathbf{E}&=\mathbf{F}\\ \mathbf{A}_{\mathrm{P}}\cdot\mathbf{E}&=\mathbf{P}\end{aligned} \qquad (1)$$

The purpose of productive components is to provide resources to other components or to obtain the final product of the system. But, components also exist, called *dissipative* components, which function is to eliminate partially or totally the wastes or residues that thrown away to the environment. Their utility lies in interacting with other components to allow a better system efficiency. In many of the cases the presence of dissipative components is essential for an energy system to operate both from a physical point of view (the induced-draft fans of a steam generator) and from a legal point of view (electrostatic precipitators for ash elimination in flue gases).

Therefore the productive structure definition must have an attribute, for each component, indicating whether it is a productive or a dissipative one. Table I shows the productive structure definition of the plant analyzed in the paper. The stack is a dissipative device whose objective is to expel combustion gases to the environment.

Table I: Productive structure of the TGAS plant

Nr	Device	Fuel	Product	Type
1	Combustor	E_5	E_2–E_1	P
2	Compressor	E_6	E_1–E_0	P
3	Turbine	E_2–E_3	E_6+E_7	P
4	HRSG	E_3–E_4	E_8	P
5	Stack	E_4	E_9	D

3. THE EXERGY COST THEORY REVISITED

As it is well known, the basis of thermoeconomic analysis is the exergy cost accounting. The exergy cost theory [11], henceforward ECT, provides a rationale for assessing the production cost in energy systems, based in terms of natural resources and their impact on the environment and helps to diagnose and optimize complex energy systems.

In this section we like to make a review of this theory, taking special interest in the computational aspects, and its application to the generation of the productive diagram.

The pure thermodynamic function called *exergy cost* is defined as follows: *Given a system whose limits, level of aggregation and production purpose of its components have been defined, we call exergy cost E_i^* of the physical flow e_i to the amount of exergy needed to produce this flow.* We call *unit exergy cost* of a flow the exergy cost per unit exergy $c_i = E_i^* / E_i$.

The exergy cost of a flow is an *emergent* property, that is, the cost value of an individual flow in a given structure is senseless, but a set of interrelated costs values, which must be determined simultaneously, does.

To find out their values, a set of propositions P1 to P5 was proposed by the ECT, see appendix in part II.

We will illustrate this procedure with the TGAS example. It has m=9 flows and n=5 units, one of them, the stack, is a dissipative unit.

The propositions from P1 to P5 offer a procedure for determining the exergy cost of the m flows of the system. Proposition P1 establishes the cost balance for each component and defines n equations. Proposition P2 provides one equation per input flow to the plant. Then the cost of the natural gas entering in the combustion chamber is known and equal to its exergy: $E_5^* = E_5$.

Note that the air entering the compressor should give another equation. However it is removed from the problem because in this case the exergy of the air could be considered as 0.

The compressor and the combustor only have an output flow then they don't generate additional equations. The fuel of the turbine has three outputs, the first one belonging to the fuel stream. According to proposition P3 one additional equation is written as:

$$\frac{E_3^*}{E_3} = \frac{E_2^*}{E_2}$$

alternatively, if we define $x_{2F} \equiv B_3 / B_2$ as the bifurcation exergy ratio corresponding to the non-exhausted fuel of the turbine the previous equation is written as:

$$-x_{3F} E_3^* + E_2^* = 0 \qquad (2)$$

The other two outputs belong to the product stream, part of the total production of the turbine comes to the compressor and the rest is part of the total product of the plant. They generate another equation according to proposition P4, and the following equation is obtained:

$$\frac{E_3^*}{E_3} = \frac{E_2^*}{E_2}$$

defining the bifurcation ratio $x_{3P} \equiv E_6 / E_6 + E_7$ the previous equation is written as:

$$-x_{3P} E_7^* + (1 - x_{3P}) E_6^* = 0 \qquad (3)$$

The last additional equation is generated in the HRSG, this fuel stream has an output and by proposition P3, we have:

$$-x_{4F} E_4^* + E_3^* = 0 \qquad (4)$$

Combining all the equations, we build the system of equations shown in Figure 1.

This system of equations could be expressed in a general form as:

$$\begin{bmatrix} \mathbf{A} \\ \boldsymbol{\alpha}_e \\ \boldsymbol{\alpha}_x \end{bmatrix} \cdot \mathbf{E}^* = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\omega}_e \\ \mathbf{0} \end{bmatrix} \qquad (5)$$

The coefficient cost matrix $(m \times m)$ is composed by:

- n rows corresponding to the incidence matrix **A**.
- ν_e rows corresponding to the definition of the system inputs, each row is made up of a

"1" in the column corresponding to the input fuel, this matrix $\boldsymbol{\alpha}_e\left(v_e \times m\right)$ satisfies:

$$\boldsymbol{\alpha}_e \cdot \mathbf{E} = \boldsymbol{\alpha}_e \cdot \mathbf{E}^* = \boldsymbol{\omega}_e \qquad (6)$$

where the vector $\boldsymbol{\omega}_e(v_e \times 1)$ contains the exergy values of the system's input.

- v_x rows corresponding to the definition the exergy distribution ratios of the system, the matrix $\boldsymbol{\alpha}_x\left(v_x \times m\right)$ satisfies:

$$\boldsymbol{\alpha}_x \cdot \mathbf{E} = \boldsymbol{\alpha}_x \cdot \mathbf{E}^* = \mathbf{0} \qquad (7)$$

This procedure will allow the definition a general algorithm that uses the exergy values of a *thermodynamic state* and the productive structure of the plant to build up the costs matrices **A**, $\boldsymbol{\alpha}_e$, and $\boldsymbol{\alpha}_x$, together with the external resources exergy vector $\boldsymbol{\omega}_e$. Equation (5) allows the simultaneous computation of the exergy cost of each flow of the plant.

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & -x_{3F} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -x_{3P} & 1-x_{3P} & 0 & 0 \\ 0 & 0 & -x_{4F} & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_1^* \\ E_2^* \\ E_3^* \\ E_4^* \\ E_5^* \\ E_6^* \\ E_7^* \\ E_8^* \\ E_9^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline E_5 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}$$

Figure 1: Exergy Cost equations of TGAS plant

Table II shows the exergy cost of the flows of the plant. The output gases, flow#9, has a cost of 860.6 kW that represents the resources dissipated in the stack. But, according to P5, the cost of flow#9 should be zero to have a correct cost account of the final products. Under these conditions, cost balance in the stack fails. Therefore, we must charged the cost of flow#9 to the final products to solve the problem. How can we allocate the residues cost if there are several products?

The Exergy Cost Theory propositions apply to productive units, but these rules fails in the case of dissipative units, and they must be extended.

The method described above has some important limitations that must be solved:

- There is not a general procedure to assess the cost of residues and to determine the effect of the residues on the production cost.
- The method computes the cost of the flows, but cannot identify the causes of the cost formation process and relates, in a straightforward way, the production cost and the irreversibility of the components.

The answers to these problems will be explained in the second part of the paper.

Table II: Exergy cost of the TGAS plant flows

Nr	E(kW)	E^* (kW)	k^*(kW/kW)
1	2765.7	5840.5	2.1118
2	9932.6	18047.3	1.8170
3	4080.1	7413.5	1.8170
4	473.6	860.6	1.8170
5	12206.8	12206.8	1.0000
6	3046.2	5840.5	1.9173
7	2500.0	4793.3	1.9173
8	2395.6	6552.9	2.7354
9	473.6	860.6	1.8170

4. THE FUEL PRODUCT TABLE

The first step in identifying the cost formation process of product and residues consists of building, from the physical structure of the plant, a productive scheme that explains the resources distribution throughout the plant. The problem of the productive structure identification is closely related to *Leontief's input-output economic analysis* [12].

An equivalent micro-economic model could be applied to energy systems. It can be represented by means of a directed graph called the *productive or functional diagram*, see ref. [13]-[14].

The graph has the same components than the physical diagram, and a new set of flows $\mathcal{G}$, such as a flow defined by the pair of components $(v_i, v_j) \in \mathcal{G}$ if the product of the component v_i becomes fuel of the component v_j. If E_{ij} represents the exergy carried out by this flow, the adjacency matrix of the graph is defined as:

$$\mathbf{G}(i,j) = \begin{cases} E_{ij} & \text{if } (v_i, v_j) \in \mathcal{G} \\ 0 & \text{otherwise} \end{cases}$$

The adjacency matrix could be represented by means of the FP table, depicted in Figure 2.

	F_0	F_1	…	F_n
P_0		E_{01}	…	E_{0n}
P_1	E_{10}	E_{11}	…	E_{1n}
…	…	…	E_{ij}	…
P_n	E_{n0}	E_{1n}	…	E_{nn}

Figure 2: Generic FP Table

According to this representation, the production of one component is used as fuel of other components or as a part of the total production of the plant:

$$P_i = E_{i0} + \sum_{j=1}^{n} E_{ij}, \quad i = 0,1,\ldots,n \tag{8}$$

In the above expression, if the component 0 is considered to be the system environment, then E_{i0} represents the production of the component *i-th* that leaves the system as a final product or as a residue. On the other hand, the resources consumed by each component could be written as:

$$F_i = E_{0i} + \sum_{j=1}^{n} E_{ji}, \quad i = 0,1,\ldots,n \tag{9}$$

where E_{i0} represents the external resources used in the *i-th* component.

Although the productive diagram could be built *manually* for a particular system, Valero and Torres [15] introduced in 1988 an algorithm to obtain general analytic formulae of cost and efficiency, which could be applied to generate the FP table for any energy system no matter how complex it is.

Let us consider the following matrix relationship:

$$\begin{bmatrix} \mathbf{A}_P \\ \boldsymbol{\alpha}_e \\ \boldsymbol{\alpha}_x \end{bmatrix} \cdot \mathbf{E} = \begin{bmatrix} \mathbf{P} \\ \boldsymbol{\omega}_e \\ \mathbf{0} \end{bmatrix} \tag{10}$$

the left side is a $(m \times m)$ regular matrix, and its inverse could be written as:

$$\left[\mathbf{A}_P^{(-1)}, \quad \boldsymbol{\alpha}_e^{(-1)}, \quad \boldsymbol{\alpha}_x^{(-1)} \right]$$

and eq. (10) becomes:

$$\mathbf{E} = \mathbf{A}_P^{(-1)}\mathbf{P} + \boldsymbol{\alpha}_e^{(-1)}\boldsymbol{\omega}_e \tag{11}$$

The term $\mathbf{B}_e \equiv \boldsymbol{\alpha}_e^{(-1)}\boldsymbol{\omega}_e = \boldsymbol{\alpha}_e^T\boldsymbol{\omega}_e$ is a $(m \times 1)$ vector, which refers to the external resources and their elements are given by:

$$B_{e,i} = \begin{cases} E_i & \text{if } e_i \in \mathcal{P}_0 \\ 0 & \text{otherwhise} \end{cases}$$

Multiplying both sides of equation (11) by the fuel incidence matrix yields:

$$\mathbf{F} = \mathbf{A}_F\mathbf{A}_P^{(-1)}\mathbf{P} + \mathbf{A}_F\mathbf{B}_e \tag{12}$$

the term $\mathbf{F}_e \equiv \mathbf{A}_F\mathbf{B}_e$ is a $(n \times 1)$ vector that indicates the exergy of the external resources entering each component.

If we define $\langle \mathbf{FP} \rangle \equiv \mathbf{A}_F\mathbf{A}_P^{(-1)}$, then eq. (12) could be written as:

$$\mathbf{F} = \mathbf{F}_e + \langle \mathbf{FP} \rangle \mathbf{P} \tag{13}$$

$\langle \mathbf{FP} \rangle$ is a $(n \times n)$ matrix whose elements y_{ij} represent the portion of the product of the *j-th* component that becomes fuel for the *i-th* component, according to eq. (9) we could identify the elements of the fuel-product table as:

$$\begin{aligned} E_{0i} &= F_{e,i} \qquad i = 1,\ldots,n \\ E_{ji} &= y_{ij}P_j \quad i,j = 1,\ldots n \end{aligned} \tag{14}$$

and substituting eq. (14) into equation (8), we get:

$$E_{i0} = \left(1 - \sum_{j=1}^{n} y_{ji} \right) \quad i = 1,\ldots,n \tag{15}$$

If the productive definition of every component and the exergy of every flow are known, then equations (14) and (15) let us build the fuel-product table of a thermodynamic state of any system, no matter how complex it is. This method provides a generic interface between thermodynamic and thermoeconomic applications.

Table III: FP table of the TGAS plant

	F_0	F_1	F_2	F_3	F_4	F_5	Σ
P_0		12207	0	0	0	0	12207
P_1	0	0	0	4223	2602	341	7166
P_2	0	0	0	1629	1004	132	2765
P_3	2500	0	3046	0	0	0	5546
P_4	2395	0	0	0	0	0	2395
P_5	473	0	0	0	0	0	473
Σ	5369	12207	3046	5852	3606	473	

Table III shows the values of the FP table for the analyzed plant. The FP table represents the adjacency matrix of the productive structure graph. Therefore the productive diagram of a plant, see Figure 3, could be drawn, using an appropriate algorithm [16]-[17], with the information provided by the FP table.

5. GUIDELINES AND CONCLUSION

In this paper the essential information required to build the thermoeconomic model of an energy system has been shown.

As a resume of the methods and procedures described above, we give a step by step guideline to obtain the Fuel-Product table. Given an energy system:

1. Provide the physical structure of the system as a collection of components and flows connecting these components, defined by means of the incidence matrix.

2. Provide a thermodynamic model that allows the computation of the exergy of each mass and energy flows defined in the physical structure of the plant.
3. Provide the productive structure for each component of the plant, given a list of the physical flows that constitute their fuel and product streams.
4. Build the matrices $\mathbf{A}_F$, $\mathbf{A}_P$, $\boldsymbol{\alpha}_e$, and $\boldsymbol{\alpha}_x$, and the exergy resources vector $\boldsymbol{\omega}_e$, with the information provide in point 2 and 3.
5. Apply the algorithm described in equations (10)-(15), to build the FP table.

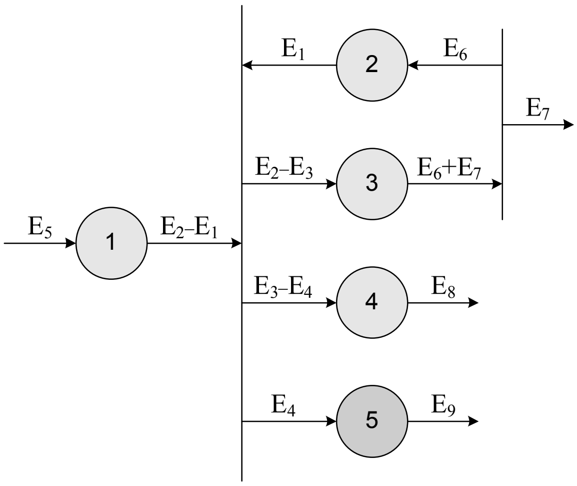

Figure 3: Productive Diagram of the TGAS plant

The productive diagram obtained from the FP table is a graphical representation of the thermoeconomic model. It plays a key role in the cost formation process analysis and thermoeconomic diagnosis, which will be explained in the second part of the paper.

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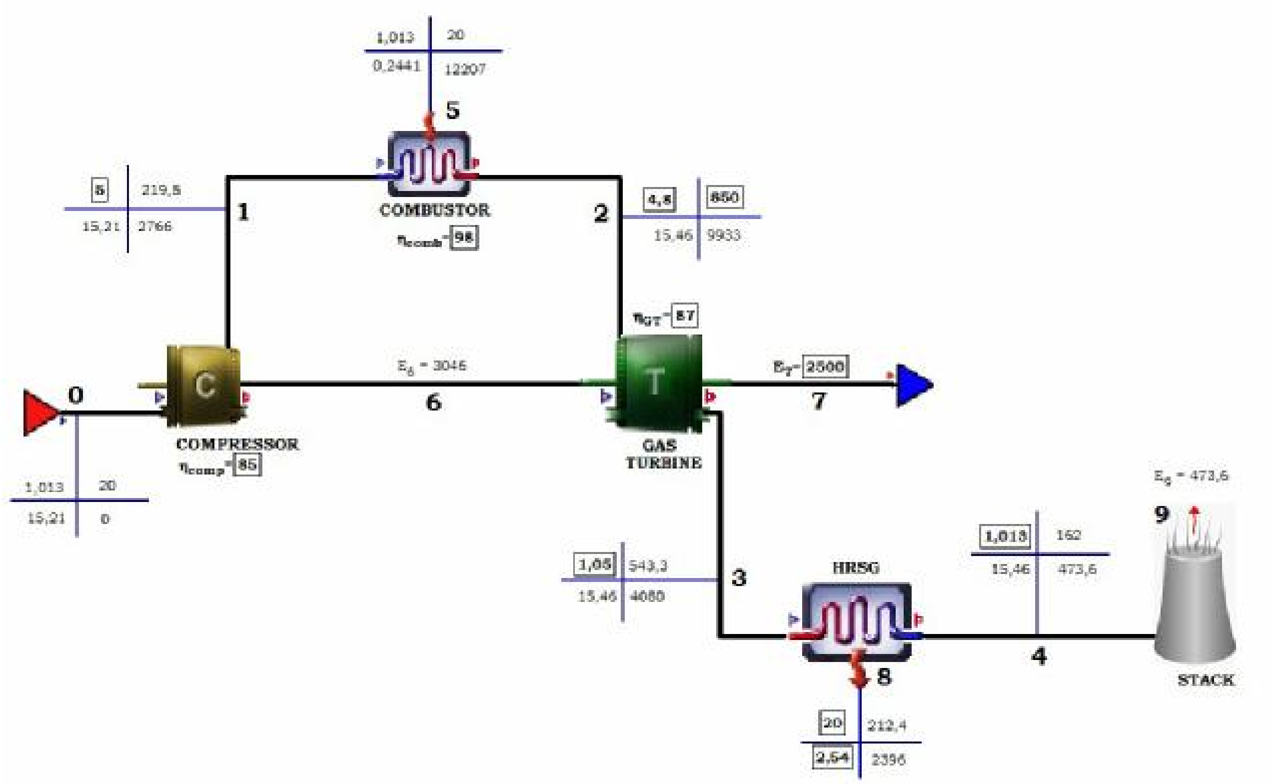

Figure 4: Flow sheet diagram of a gas turbine cogeneration plant

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Part II: Application to Thermoeconomic Diagnosis

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ABSTRACT: The aim of this second part of the paper is to show the capabilities of the thermoeconomic analysis to the diagnosis of energy systems. A collection of algorithms for the analysis of the cost formation process of products and residues is presented. They use the FP table, a mathematical representation of the thermoeconomic model discussed in the first part of the paper, as input data. These results are applied to the thermoeconomic diagnosis of energy systems. Thermoeconomic diagnosis is performed by comparing two states of the system a current state that one wants to analyze and a reference state with which one wants to compare with. The algorithms to compute the impact of the malfunctions in terms of additional resources consumption and the analysis of the causes of the increase of irreversibilities and residues are described.

Keywords: Thermoeconomics, Residues, Cost Accounting, Diagnosis.

NOMENCLATURE

n	Number of components
E	Exergy of a flow [kW]
F	Fuel exergy of a component [kW]
P	Product exergy of a component [kW]
I	Irreversibility of a component [kW]
c	Unit exergoeconomic cost [kW/kW]
F_T	Exergy of the external resources [kW]
C_F	Exergy cost of Fuel [kW]
C_P	Exergy cost of Product [kW]
C_R	Exergy cost of Residue [kW]
C_e	Exergy cost of external resources [kW]

Matrices and Vectors

$\mathbf{c}_P$	Unit production cost vector (n × 1)
$\mathbf{c}_R$	Unit exergy cost of residues vector (n × 1)
$\boldsymbol{\omega}_0$	System outputs exergy vector (n × 1)
$\boldsymbol{\omega}_s$	Final product exergy vector (n × 1)
$\boldsymbol{\omega}_r$	Residues exergy vector (n × 1)
$\mathbf{u}$	Unity vector (n × 1)
$\mathbf{U}_D$	Identity Matrix (n × n)
$\mathbf{K}_D$	Diagonal matrix which contain the unit consumption of each component (n × n)
$\langle\mathbf{KP}\rangle$	Matrix (n × n) which contains the unit consumption
$\langle\mathbf{KR}\rangle$	Matrix (n × n) which contains the residue ratios
$\lvert\mathbf{P}\rangle$	Product operator matrix (n × n)
$\lvert\mathbf{I}\rangle$	Irreversibility operator matrix (n × n)
$\lvert\mathbf{R}\rangle$	Irreversibility operator matrix (n × n)
MF	Malfunctions vector (n × 1)
$\mathbf{MF}^*$	Malfunctions vector (n × 1)
[**MF**]	Malfunction matrix (n × n)
DF	Dysfunctions vector (n × 1)
[**DF**]	Dysfunction matrix (n × n)

Greek letters

κ	Unit exergy consumption
ψ	Residue cost distribution ratios
ρ	Residue generation per unit of production

Δ	Increment

Sets

$\mathcal{V}_P$	Productive components of a system
$\mathcal{V}_D$	Dissipative components of a system

Subscripts and superscripts

e	Related to external resources
s	Related to final product
r	Related to residues
*	Exergy cost
t	Transpose matrix
-1	Inverse matrix
0	Reference conditions

1. INTRODUCTION

Thermoeconomic analysis combines economic and Second Law analysis and it helps to point out how, where and what amount of resources are consumed and indicates what improvements could be made. Performed during the life of an operating plant it can reveal the efficiency degradation of the plant and its components.

Thermoeconomic analysis is used, among other applications, for:

- Assess rational prices of the plant production based on physical criteria.
- Optimize specific process variables to minimize the production cost of the system.
- Compare several design alternatives or operation decisions.
- Detect inefficiency increases and the calculation of their economic impact.

Let us remember that Exergy Cost Theory and other cost accounting methodologies, see [1], are based on cost allocation rules, which attribute to the useful product of each component the resources required to obtain it, and distribute its cost proportionally to exergy. They are numerical techniques that calculate the values in an accurate way, but they lack a mathematical structure. It is not easy to identify the cost formation process of products and residues. To do this, it is not enough to calculate the exergy cost, but to provide general equations that relate the production costs with the efficiency and irreversibilities of each individual component of the system.

In the case of a sequential system, where the product of a process is the fuel of the next one, the unit exergy cost of the product (kW/kW) of the *i-th* process could be written as a function of the unit consumption of each individual component:

$$c_{P,i} = k_1 \cdot k_2 \cdots k_i$$

or as a function of the irreversibilities generated in the processes required to obtain it:

$$c_{P,i} = 1 + \sum_{r=1}^{i} I_r / P_i$$

And the increment of resources consumptions, due to the increment of the unit consumption of each component is:

$$\Delta F_T = \sum_{i=1}^{n} c_{F,i} \Delta k_i P_i^0$$

Symbolic Exergoeconomics [2] provides relationships that allows generalize the expressions mentioned above to any energy system.

Just as it has been explained in the first part, besides the analysis of the cost formation process of products, it is also necessary to identify the formation process of *residues* [3] to make a correct assessment of the residue cost to the productive units, based on the principle "*Those who produce (pollute) should pay for it*".

These subjects will be explained in this paper, and illustrated with the TGAS plant described in previous paper.

2. THE PRODUCTIVE STRUCTURE

The thermoeconomic model, introduced in the first part of the paper, provides a productive scheme that could be represented by means the FP table. In this model, the inputs of each component are their resources or fuels and the outputs are their products. These products become fuel of other components, part of the final product or they are eliminated in the dissipative units. Reciprocally the fuel of a component comes from external resources or from the product of other component. Each element of the FP table, denoted by E_{ij}, represents the exergy of the product of the component v_i becomes fuel of the component v_j.

In the first part of the paper an algorithm to obtain the productive diagram and the values of the FP table for each thermodynamic state of a plant was described through. Next, we will explain step by step how to compute production costs and to obtain equations that allow the explanation of the cost formation process of products and residues.

In the case of the productive units of the system, the product could be expressed as:

$$P_i = E_{i0} + \sum_{j=1}^{n} E_{ij}, \quad i \in \mathcal{V}_P \tag{1}$$

and for the dissipative units, it is written as:

$$P_i = R_{i0}, \quad i \in \mathcal{V}_D \tag{2}$$

where R_{i0} represents the exergy dissipated in the *i-th* component. The fuel of each component is given by:

$$F_i = E_{0i} + \sum_{j=1}^{n} E_{ji}, \quad i \in \mathcal{V} \tag{3}$$

dividing both terms of eq. (3), by P_i, we could decompose the unit exergetic cost of each component into the sum of the contribution of the unit consumption of each individual resource:

$$k_i = \kappa_{0i} + \sum_{j=1}^{n} \kappa_{ij} \tag{4}$$

where $\kappa_{ij} = E_{ij} / P_j$ is the amount of resources coming from the *i-th* component required to obtain a unit of the product of the *j-th* component, then eq. (1) could be rewritten as:

$$P_i = E_{i0} + \sum_{j=1}^{n} \kappa_{ij} P_j \tag{5}$$

According to the Structural Theory [4], this equation represents the characteristic equation of the Thermoeconomic Model (TEM). It is write in matrix notation as follows:

$$\mathbf{P} = \boldsymbol{\omega}_0 + \langle \mathbf{KP} \rangle \mathbf{P} \tag{6}$$

where $\langle \mathbf{KP} \rangle$ is a ($n \times n$) matrix, whose elements are the unit exergy consumption κ_{ij} related to the internal resources of the system, and $\boldsymbol{\omega}_0 (n \times 1)$ is a vector whose elements are the outputs of the system, both final products $\boldsymbol{\omega}_s$, in the case of productive units or residues $\boldsymbol{\omega}_r$, in case of dissipative units, which satisfy:

$$\boldsymbol{\omega}_0 = \boldsymbol{\omega}_s + \boldsymbol{\omega}_r \tag{7}$$

Equation (6) allows us to relate the production of each component as a function of unit consumption of each component and the system outputs:

$$\mathbf{P} = |\mathbf{P}\rangle \boldsymbol{\omega}_0 \tag{8}$$

where $|\mathbf{P}\rangle \equiv (\mathbf{U}_D - \langle \mathbf{KP} \rangle)^{-1}$ is a ($n \times n$) matrix, called *production operator*, which is a key element of the FP model.

Similarly we could define the irreversibility operator: $|\mathbf{I}\rangle \equiv (\mathbf{K}_\mathrm{D} - \mathbf{U}_\mathrm{D})|\mathbf{P}\rangle$ that verifies:

$$\mathbf{I} = |\mathbf{I}\rangle \boldsymbol{\omega}_0 \tag{9}$$

An appropriate way to compute these operators is the LU decomposition method, described for example in reference [5].

3. COST EQUATIONS MODEL

Once the basic elements of the productive structure have been presented, we will compute the exergy cost of the productive structure flows E_{ij}^*. In accordance with the proposed model the cost of the resources used in each component is given by:

$$C_{F,i} = E_{i0}^* + \sum_{j=1}^{n} E_{ji}^* \quad v_i \in \mathcal{V} \tag{10}$$

where E_{0i}^* represents the cost of the external resources entering the component. Likewise, the cost of the productive units is:

$$C_{P,i} = E_{0i}^* + \sum_{j=1}^{n} E_{ij}^* \quad v_i \in \mathcal{V}_P \tag{11}$$

where E_{i0}^* represents the cost of the final products of the system. In the case of dissipative units' yields:

$$C_{P,i} = R_{i0}^* \quad i \in \mathcal{V}_D \tag{12}$$

where R_{i0}^* represents the cost of the output of the dissipative unit.

Torres et al. [3] proposed that the cost of a residue R_{i0}^* can be broken down into several costs one for each component:

$$R_{i0}^* = \sum_{v_j \in \mathcal{V}_P} R_{ij}^* \quad i \in \mathcal{V}_D \tag{13}$$

R_{ij}^* is assumed as the cost of the dissipated residue by the *i-th* component that has been produced by the *j-th* component. Therefore, the cost of the residues charged to the *j-th* productive component is given by:

$$C_{R,j} = \sum_{v_i \in \mathcal{V}_D} R_{ij}^* \quad v_j \in \mathcal{V}_P \tag{14}$$

In order to determine the values of R_{ij}^*, a distribution cost ratio ψ_{ij} must be defined such as:

$$R_{ij}^* = \psi_{ji} R_{i0}^* \quad \text{and} \quad \sum_j \psi_{ji} = 1 \tag{15}$$

However, there is not a general criterion to define the residue cost distribution ratios, and it depends of the type of residue and dissipative unit. A criterion that has been frequently used, is to allocate the cost of residues proportionally to the entropy generated during the process, see [6]-[7]. This criterion works for closed cycles, like Rankine or refrigeration cycles, but fails for other types like gas turbines.

We propose a simple method to define the cost distribution ratios of the residues, that makes them proportional to the exergy of the flows processed in the dissipative unit:

$$\psi_{ji} = E_{ji} / F_i \qquad (16)$$

The main advantage of this criterion is that the coefficients could be obtained directly from the information provided by the FP table. In other cases this information should be provided by the user. It is important to remark that this option simplifies the software implementation and could be used as a default option, but we should clarify that it is neither the only way nor the best option for all systems. The cost distribution ratios for the TGAS example, corresponding to the gases expelled by the stack are shown in Table I. According to these criteria 72% of the residual gases cost is charged to the combustor and the rest to the compressor.

Table I: Residue cost distribution ratios for the combustion gases on the TGAS plant

	1	2	3	4	5
ψ_5	0.7216	0.2784	0	0	0

One of the most interesting features of the FP model is that the ECT rules, see appendix, could be simplified, as follow:

- According to propositions P1 and P5, the cost of the product of each component is equal to the cost of the resources required to obtain it, plus the cost of the residues produced.

$$C_{P,i} = C_{F,i} + C_{R,i} \qquad (17)$$

- In accordance with proposition P2, the cost of the external resources, namely $C_{e,i}$, must be known. The exergy cost of each flow entering the plant is equal to its exergy:

$$C_{e,i} \equiv E^*_{i0} = E_{i0} \qquad (18)$$

- Regarding propositions P3 and P4, the cost of each flow making up the product of a component is proportional to its exergy:

$$E^*_{ij} = c_{P,i} E_{ij} \qquad (19)$$

Combining equations (10),(14),(18) and (19) into equation (17) yields:

$$c_{P,i} = c_{e,i} + \sum_{j=1}^{n} c_{P,j} (\kappa_{ji} + \rho_{ji}) \qquad (20)$$

where $c_{e,i} \equiv C_{e,i} / P_i$ is the cost of external resources per unit produced, and $\rho_{ji} = \psi_{ij} P_j / P_i$ represents the ratio of production of the *j-th* component that becomes residue dissipated in the *i-th* component, and satisfies:

$$\omega_{r,i} \equiv R_{i0} = \sum_{v_j \in V_P} \rho_{ij} P_j \quad v_i \in \mathcal{V}_D \qquad (21)$$

Expression (20) computes simultaneously the production cost of each component by solving the system of linear equations:

$${}^t\mathbf{c}_P \left(\mathbf{U}_D - \langle \mathbf{KP} \rangle - \langle \mathbf{KR} \rangle \right) = {}^t\mathbf{c}_e \qquad (22)$$

where $\langle \mathbf{KR} \rangle$ is a ($n \times n$) matrix, which contains the ρ_{ij} coefficients. Note that the rows of the $\langle \mathbf{KR} \rangle$ matrix corresponding to the productive units are zero, meanwhile the rows of the $\langle \mathbf{KP} \rangle$ corresponding to the dissipative units are also zero.

Table II: FP Cost table for TGAS plant (kW)

	$C_{F,0}$	$C_{F,1}$	$C_{F,2}$	$C_{F,3}$	$C_{F,4}$	$C_{F,5}$	Σ
$C_{P,0}$		12207	0	0	0	0	12207
$C_{P,1}$	0	0	0	7618	4648	663	12930
$C_{P,2}$	0	0	0	3888	2372	339	6599
$C_{P,3}$	5187	0	6320	0	0	0	11507
$C_{P,4}$	7020	0	0	0	0	0	7020
$\mathbf{C}_F$	12207	12207	6320	11507	7020	1002	
$C_{P,5}$	0	723	279	0	0	0	1002
$\mathbf{C}_R$	0	723	279	0	0	0	

Once the production cost of every component is obtained, it is possible to build the corresponding FP cost table, which contains the cost of the productive structure flows, computed by means of eq. (19) and the residue costs R^*_{ij} that could be calculated as:

$$R^*_{ij} = c_{P,i} \rho_{ij} P_j \qquad (23)$$

The FP cost table for the TGAS example is shown in Table II. In this example the residue costs represent 8% of the total cost of the resources.

If the cost of the external resources takes also into account the market price of the consumed fuel c_i (c/kWh), the investment and operation cost of the equipment as well as the abatement cost of residues Z_i (€/h), then the proposed methodology could extended to compute exergoeconomic costs [1], substituting eq. (18) by:

$$C_{e,i} = c_i E_{i0} + Z_i \tag{24}$$

4. THE FORMATION PROCESS OF PRODUCTS AND RESIDUES

In the previous section an equation to compute the production cost including the residue impact has been obtained. Now, we will analyze in depth the cost formation process of product and residues.

Equation (22) could be reordered as:

$${}^t\mathbf{c}_P\left(\mathbf{U}_D - \langle \mathbf{KP} \rangle\right) = {}^t\mathbf{c}_e + {}^t\mathbf{c}_R \tag{25}$$

where $\mathbf{c}_R \equiv {}^t\langle \mathbf{KR} \rangle \mathbf{c}_P$ represents the cost of the residues per unit produced in each component.

Then, the previous equation allows the decomposition of the production cost as:

$$\mathbf{c}_P = {}^t|\mathbf{P}\rangle\left(\mathbf{c}_e + \mathbf{c}_R\right) = \mathbf{c}_P^e + \mathbf{c}_P^r \tag{26}$$

where:

$$\mathbf{c}_P^e = {}^t|\mathbf{P}\rangle \mathbf{c}_e \tag{27}$$

represents the unit exergy cost of the product due to the contribution of the external resources, and only depends on the local unit consumption of the components. Meanwhile,

$$\mathbf{c}_P^r = {}^t|\mathbf{P}\rangle \mathbf{c}_R \tag{28}$$

represents the unit exergy cost due to residue cost that depends on the choice of the residue distribution ratios.

The exergy cost of products due to the external resources could be decomposed by the contributions due to the irreversibilities generated along the process. From eq.(4) the following relationship states:

$${}^t\mathbf{c}_e = {}^t\mathbf{u}\left(\mathbf{K}_D - \langle \mathbf{KP} \rangle\right) \tag{29}$$

And substituting it into eq. (27) we get:

$${}^t\mathbf{c}_P^e = {}^t\mathbf{u}\left(\mathbf{K}_\text{D} - \mathbf{U}_\text{D}\right)|\mathbf{P}\rangle = {}^t\mathbf{u}\left(\mathbf{U}_\text{D} + |\mathbf{I}\rangle\right) \tag{30}$$

Hence, the exergy cost of products could be decomposed into the cost due to the contribution irreversibility of each component and the contribution of each residue.

If we define $|\mathbf{R}\rangle \equiv \langle \mathbf{KR} \rangle |\mathbf{P}\rangle$ as the residue operator eq. (26) could be expressed as:

$$\mathbf{c}_P^r = {}^t|\mathbf{R}\rangle \mathbf{c}_P \tag{31}$$

The $|\mathbf{R}\rangle$ matrix has only got rows corresponding to dissipative units different to zero, then eq. (31) provides a decomposition of the production cost due to each residue:

$$c_{P,i}^r = \sum_{j \in \mathrm{V_D}} r_{ji} c_{P,i} \tag{32}$$

The elements r_{ij} of the $|\mathbf{R}\rangle$ matrix represent the portion of the unit production cost of the *i-th* component due to the residue dissipated in the *j-th* component.

It is important to remark that the production and residues costs depend on the defined aggregation level. For more accurate results flow decomposition into mechanical, thermal and chemical exergy components is recommended, see [8].

Figure 1 shows a graph of the exergy cost of the TGAS plant components, including the effect of the irreversibilities of the components involved in the production process and the residues.

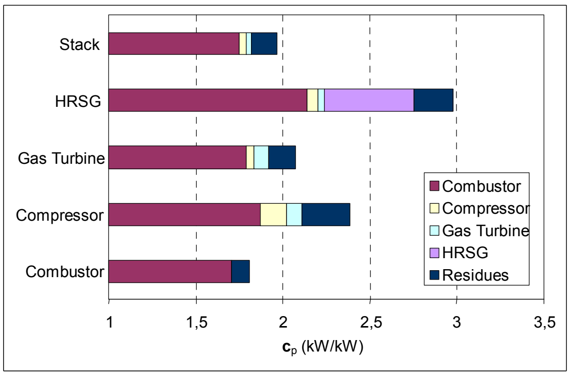

Figure 1: Cost Decomposition graph

If we have first compute the unit production cost due to the internal irreversibilities $\mathbf{c}_P^e$, substituting eq. (31) into eq. (26), the unit exergy cost of the product $\mathbf{c}_P$ could be obtained by solving the system of linear equations:

$$\left(\mathbf{U}_\text{D} - {}^t|\mathbf{R}\rangle\right)\mathbf{c}_P = \mathbf{c}_P^e \tag{33}$$

This result provides a step by step procedure to compute the exergy cost decomposition:

1. Compute the unit exergy consumption of each component: $\mathbf{K}_\text{D}, \langle \mathbf{KP} \rangle$
2. Compute the product operator:
$$|\mathbf{P}\rangle = \left(\mathbf{U}_\text{D} - \langle \mathbf{KP} \rangle\right)^{-1}$$
3. Compute the irreversibility operator:
$$|\mathbf{I}\rangle = \left(\mathbf{K}_D - \mathbf{U}_D\right)|\mathbf{P}\rangle$$
4. Compute the exergy cost of products due to the irreversibilities:
$${}^t\mathbf{c}_P^e = {}^t\mathbf{u}\left(\mathbf{U}_D + |\mathbf{I}\rangle\right)$$

5. Compute the residue operator:

$$|\mathbf{R}\rangle = \langle\mathbf{KR}\rangle|\mathbf{P}\rangle$$

6. Compute the expression:

$$\mathbf{U}_D + |\tilde{\mathbf{R}}\rangle \equiv \left(\mathbf{U}_D - |\mathbf{R}\rangle\right)^{-1}$$

7. Compute the exergy cost of product:

$${}^t\mathbf{c}_P = {}^t\mathbf{c}_P^e\left(\mathbf{U}_D + |\tilde{\mathbf{R}}\rangle\right)$$

8. Compute the exergy cost of products due to residues:

$$\mathbf{c}_\mathrm{P}^r = {}^t|\mathbf{R}\rangle\mathbf{c}_P = {}^t|\tilde{\mathbf{R}}\rangle\mathbf{c}_\mathrm{P}^e$$

We like to note that the exergy cost balance of the total system could be written in two ways:

$$F_T = {}^t\mathbf{c}_P\boldsymbol{\omega}_s = {}^tc_P^e\left(\boldsymbol{\omega}_s + \boldsymbol{\omega}_r\right) \qquad (34)$$

In the first way the exergy cost of the external resources are exclusively distributed to the final product proportional to the unit production cost. The second way considers all outputs, both final products and residues valuated at the unit production cost due to the irreversibilities.

Table III shows a resume of the unit exergy cost decomposition of each component.

Table III: Production Cost for TGAS (kW/kW)

Dev	k	$\mathbf{c}_P^e$	$\mathbf{c}_P^r$	$\mathbf{c}_P$
1	1.7032	1.7032	0.0895	1.7927
2	1.1014	2.1114	0.2433	2.3548
3	1.0551	1.9170	0.1396	2.0566
4	1.5076	2.7391	0.1995	2.9386
5	1.0000	1.8169	0.1324	1.9492

5. APPLICATION TO THERMOECONOMIC DIAGNOSIS

One of the most important applications of thermoeconomic analysis is the location and quantification of the anomalies causing the reduction of the system efficiency.

Thermoeconomic diagnosis is always performed through the comparison of two thermodynamic states. The presence of anomalies in the actual state is determined by the deviation in some parameters and indexes with respect to a reference state. This method could provide both the operator and the plant manager with useful information about the possibility of delaying unscheduled repairs, of reducing the output, of changing the energy mix, etc.

The basis of the thermoeconomic diagnosis is to relate the variation of the irreversibilities and the total resources consumption of the plant under study to the variation of the efficiency of each component of the plant.

The objective of this section is to review the calculation of the most important indexes described in [10], to include the effect of residues and provide the algorithms to compute them. We will use the matrix notation because it is more convenient for software implementation.
The variation of the resources consumption could be expressed as the sum of the irreversibility variation, and system output variation.

$$\Delta F_T = {}^t\mathbf{u}\Delta\mathbf{I} + {}^t\mathbf{u}\Delta\boldsymbol{\omega}_0 \qquad (35)$$

The irreversibility variation of each component could be decomposed into two contributions, as Valero states in [9]:

$$\Delta\mathbf{I} = \Delta\mathbf{K}_\mathrm{D}\mathbf{P}^0 + \left(\mathbf{K}_\mathrm{D} - \mathbf{U}_\mathrm{D}\right)\Delta\mathbf{P} \qquad (36)$$

- The malfunction due to the variation of the efficiency of the component itself:

$$\mathbf{MF} = \mathbf{K}_\mathrm{D}\mathbf{P}^0 \qquad (37)$$

- The dysfunction due to the variation of the production objective of the component:

$$\mathbf{DF} = \left(\mathbf{K}_\mathrm{D} - \mathbf{U}_\mathrm{D}\right)\Delta\mathbf{P} \qquad (38)$$

The variation of the product of each component could be expressed also as a function of the unit consumption and system's outputs variation:

$$\Delta\mathbf{P} = |\mathbf{P}\rangle\Delta\langle\mathbf{KP}\rangle\mathbf{P}^0 + |\mathbf{P}\rangle\Delta\boldsymbol{\omega}_0 \qquad (39)$$

then the dysfunction relation (38), could be written as:

$$\mathbf{DF} = |\mathbf{I}\rangle\Delta\langle\mathbf{KP}\rangle\mathbf{P}^0 + |\mathbf{I}\rangle\Delta\boldsymbol{\omega}_0 \qquad (40)$$

The dysfunction produced in a component is caused by the malfunctions in other components. Therefore it could be interesting to decompose the dysfunction vector in the contribution of each component. For this purpose we define $[\mathbf{MF}] \equiv \left(\Delta\kappa_{ij}P_j^0\right)_{i,j=1,\ldots,n}$ as the malfunction matrix and $\mathbf{MF}_e \equiv \left(\Delta\kappa_{0j}P_j^0\right)_{j=1,\ldots,n}$ as the malfunction vector of the external resources, therefore:

$$\mathbf{MF} = \mathbf{MF}_e + [\mathbf{MF}]\mathbf{u} \qquad (41)$$

In the same way we define the dysfunction matrix as $[\mathbf{DF}] \equiv |\mathbf{I}\rangle[\mathbf{MF}]$ and the dysfunction caused by the variation of the system output as the vector $\mathbf{DF}_0 \equiv |\mathbf{I}\rangle\Delta\boldsymbol{\omega}_0$, therefore:

$$\mathbf{DF} = [\mathbf{DF}]\mathbf{u} + \mathbf{DF}_0 \qquad (42)$$

The dysfunction matrix lets us know the contribution of each component to the irreversibility of other components, so DF_{ij} represents the increase of the irreversibility of the *i-th* component due to the malfunction of the *j-th* component.

Then, substituting eq.(41) and (42) into eq. (36) we got :

$$\Delta \mathbf{I} = \mathbf{MF} + [\mathbf{DF}]\mathbf{u} + \mathbf{DF}_0 \quad (43)$$

Applying eq (43) to total fuel variation, equation (35), the well-known fuel impact formula is obtained:

$$\Delta F_T = {}^t\mathbf{uMF}_e + {}^t\mathbf{c}_P^e[\mathbf{MF}]\mathbf{u} + {}^t\mathbf{c}_P^e\Delta\boldsymbol{\omega}_0 \quad (44)$$

From the fuel impact formula, we can define an important index, called malfunction cost, which indicates the increase in the amount of resources caused by an internal malfunction, and is given by:

$${}^t\mathbf{MF}^* = {}^t\mathbf{MF}_e + {}^t\mathbf{c}_P^e[\mathbf{MF}] \quad (45)$$

On the other hand, the variation of the system outputs has two parts as it is expressed in the following equation:

$$\Delta\boldsymbol{\omega}_0 = \Delta\boldsymbol{\omega}_s + \Delta\boldsymbol{\omega}_r \quad (46)$$

The variation of the final products $\Delta\boldsymbol{\omega}_s$, that could be fixed by the analyst, and the variation of the residues $\Delta\boldsymbol{\omega}_r$ that depends on the malfunctions of other components. In reference [11] a different approach that expresses the irreversibility variation as a function of the variation of the unit consumptions and the cost distribution ratios is presented.

Therefore eq. (44) could be also written as:

$$\Delta F_T = {}^t\mathbf{uMF}^* + {}^t\mathbf{c}_P^e\Delta\boldsymbol{\omega}_s + {}^t\mathbf{c}_P^e\Delta\boldsymbol{\omega}_r \quad (47)$$

It is necessary to remark that the malfunctions cost are valuated at the production cost due to the irreversibilities $\mathbf{c}_P^e$, meanwhile the fuel impact caused by the residues is given by the term ${}^t\mathbf{c}_P^e\Delta\boldsymbol{\omega}_r$. Meaning that the fuel input formula doesn't depend on the residues ratios assessment.

In summary of this section, a step by step guide for thermoeconomic diagnosis is made:

1. Select two thermodynamic states, one as reference and the other as the actual one.
2. Compute the cost analysis for both states. The results of the cost analysis obtained in section 4, could be called the *thermoeconomic state of the system.*
3. Build the malfunction vector and matrices: **MF**, [**MF**], $\mathbf{MF}_e$
4. Compute the dysfunction vector and matrices: **DF**, [**DF**], $\mathbf{DF}_s$.
5. Build the irreversibility increase table that decomposes the irreversibilities in malfunction and dysfunctions, see eq. (43)
6. Compute the malfunction cost vector $\mathbf{MF}^*$ and the residues cost variation ${}^t\mathbf{c}_P^e\Delta\boldsymbol{\omega}_s$, it allowing the decomposition of the total fuel variation, as it is shown in eq. (47)

Table IV: Thermoeconomic Diagnosis of the TGAS Plant

Dev	ΔI	$\Delta\boldsymbol{\omega}_r$	MF	MF^*	$\mathbf{c}_P^e\Delta\mathbf{P}_R$
1	51.88	0.00	-11.85	-11.85	0.00
2	26.01	0.00	19.93	38.31	0.00
3	4.53	0.00	0.00	3.77	0.00
4	18.72	0.00	18.72	36.40	0.00
5	0.00	41.59	0.00	0.33	75.78
Total	101.14	41.59	26.80	66.96	75.78

As an example of the functionality described in the paper, we analyze a decrease of 1% in the isentropic efficiency of the compressor keeping constant the production of electricity and steam. Table IV shows the main indexes of thermoeconomic diagnosis and Figure 2 represents a graph that decompose the irreversibility increase as the contribution of the own malfunctions and the malfunction generated by other components and the residues increase. Note that the cost of the residues increase is bigger than the cost of the internal malfunctions.

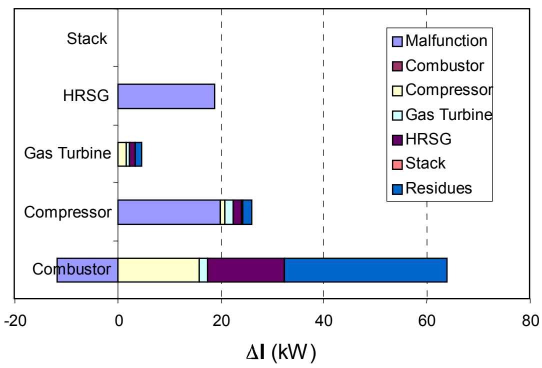

Figure 2: Irreversibility increase analysis of the TGAS plant

6. CONCLUSIONS

In the second part of the paper we have introduce the equations to compute and decompose the exergetic cost and they have been applied to obtain thermoeconomic diagnosis indexes. The paper places special emphasis on the process of cost formation of residues and its effect on the production costs. The problem of the residue cost formation was treated in the original works of the ECT [12], but it has been reviewed in recent years and is a current research line, see [3] and [11].

The full paper should be understood as a revision of the fundamental concepts of the thermoeconomic diagnosis and to serve as a functional description of the requirements for developing software for the thermoeconomic analysis of energy systems. The functional description of the requirements is the first and probably most important phase on the software development. It will serve to define an application program interface (API) with all the required data structure definitions and algorithms. The matrix formulation used in the paper is intended to be used in general programming languages as FORTRAN, C++, or Java and numerical analysis applications as MathLab.

The next step is to develop an application that could be integrated with existing computer aided applications for energy systems, just like a "plug-in" or a "standalone" application that allows to import information for a *thermodynamic state* of the system and its economic model. It should provide a graphic user interface to:

- Define the productive structure of the system for the physical model.
- Get the *thermoeconomic state* of the plant represented for the FP table from a thermodynamic state.
- Manage several thermoeconomic states: create, modify, delete and combine for thermoeconomic diagnosis.
- Provide a report generator tool that shows in an adequate form all the cost parameters, like the figures and tables included in the paper.

From now on the problem of thermoeconomic diagnosis is not to compute cost indexes but analyze the cost results, especially when studying historical data of a given running plant or thought experiments of simulation models, like the effect of control system variables, environmental conditions, fuel quality, layout changes, and so on.

A demo program called TAESS (Thermoeconomic Analysis of Energy Systems Software) is available from the authors in [13] to illustrate the ideas presented in the paper.

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APPENDIX

Exergy Cost Theory Propositions

P1. The exergy cost is a conservative property, and therefore, for each component the cost of the inlets is equal to that of the outlets. In matrix form, the exergy cost balance for all components of the plant is written as:

$$\mathbf{A} \cdot \mathbf{E}^* = \mathbf{0}$$

P2. The exergy cost is relative to the limits of the system. In absence of external assessment, the exergy cost of each flow entering the plant is equal to its exergy. If $\mathcal{P}_0$ is the set of flows entering the system, this allows us to formulate as many equations ν_e, as flows entering the plant:

$$E_i^* = E_i \quad i \in \mathcal{P}_0$$

In order to calculate the cost of each of the m flows of the system, it will be necessary to write m independent equations. If all components have only an output flow, the system is sequential and then the problem is solved by applying the stated propositions. In other case, for each component additional equations must be written equal to the number of output flows minus one. For any structure the number of bifurcations ν_x is determined by the following relationship:

$$\nu_x = \sum_{i=1}^{n} (s_i - 1) = m - \nu_e - n$$

The additional propositions are defined as follows, and use the exergy to distribute the cost in the bifurcations:

P3. If a fuel stream of this component has an output flow (non-exhausted fuel), its unit exergy cost is the same as that of the input flow of this fuel stream.

P4. If a component has several product streams the same unit exergy cost will be assigned to the product of all of them. If the product stream of a component has several output flows, the same unit exergy cost will be assigned to all of them.

P5. All the costs generated by the productive process, including the cost of residues and wastes disposal must be assessing to the cost of the final products.